Why are we not allowed to use rational agent models as the instrument to recount the behavior in preference space?

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Abstracts:

The ICP (indeterminate, coincidental, path-dependent) property of the preference behavior, which the human cognitive system entails, forbids the use of function mapping from goods space to preference space. It denies the compliance with the compactness in preference space. The adoption of functional relations as instruments to describe decisions in the preference space is blocked. Also, the completeness of the preference space is untenable due to the indeterminate behavior in the preference space. It forbids the metrizability of the preference space.

Keywords: indeterminate behavior, function mapping, compact, complete, metrizability

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I. Introduction

It took long years and ardent endeavor of economists to build the current architecture of economics, especially the rational agent model(RAM hereafter). The excellence of the model is revealing by the integrated structure of logic, which begins with the decision theory of individual choice. The model adopts the price as an information hub. The information hub receives the signals of resource allocation from every corner of the economy and translates and transmits them to other parts of the economy. The analytical preeminence of the model suffices to ensconce it in the center stage as mainstream economics.

However, it is also true that the challenge from the mismatch between theory and reality was relentless. The placement of basic postulates does not seem to match with reality. Although the rationality is the commanding logic of the rational agent model, the facts of life comply with the tenet of bounded rationality. The raising of this issue becomes particularly more acute since the experiments of behavioral studies unfolding conflicting outcomes with the elementary postulates of the rational agent model(Kahneman 2003).

The bounded rationality model(BRM in short) has yet to come up; the difference of the postulates between two tenets of economics suffices to reveal the contrast between theory and reality. The rationality postulate lays the basis to build the functional mapping of the rational choice theory of individuals, which becomes the foundation of the architecture of the RAM. However, it is quite astonishing how such unrealistic postulate has not been challenged seriously for the long history of economics.

This paper intends to raise a question on how the RAM maps up the modeling and pin down the problems which make the launching an alternative approach compelling. That is the upbringing of the BRM, which was triggered by the pioneering experiments of behavioral studies. Perhaps, it is the human cognitive system that behavioral studies highlighted to the attention of economists. The introduction of the human cognitive system seems to mean a revamping work to decision-making modeling.

II. Human cognitive system

Perhaps, it is not an exaggeration to say that decision theory is the centerpiece of economics. The state variables like goods combine with preference ordering, e.g., well-ordered, to build the consumer choice theory, the architecture of which is the backbone of the RAM. The process of combining preference ordering with optimization algorithm enables the mapping relation between state variables(goods) and price, i.e., demand function.

However, the experiments of behavioral studies complicated the decision theory of the RAM. The turnouts of the behavioral experiments did not support the well-ordered preference ordering of the RAM. In behavioral approaches, the human cognitive system replaced the role of preference ordering theory of the RAM. Behavioral studies revealed that the conduit connecting the price to decision-making on the choice of goods becomes blockaded by the human cognitive system's interruption.

In the behavioral approaches, the human cognitive system(HCS in short) begins with the perception. The perception promptly hit on the HCS 1 to pop the intuition up(Stanovich and West 2000). It is an immediate process, which stimulates the HCS and makes it ready to affect the steps of decision making. In contrast with the immediate process, a time-bearing process of the cognitive system works concurrently. It is the cognitive process of reasoning, labeled HCS 2. The dual systems work as the human cognitive system(Chaiken and Trope, 1999; Gilbert, 2002; Sloman, 2002; Stanovich and West, 2002).

Once perceptions are gained, they leave impressions and form percepts. Such impressions and percepts develop conceptual representations during the process of reasoning as they repeat. Percepts, as well as the conceptual representations, are evoked to affect the intuition and reasoning every now and then(Kahneman, 2003).

Such an HCS process invites a significant change in the decision-making system of individuals. Percepts or conceptual representations function as reference-point in the works of the cognitive system. Perception becomes reference-dependent(Kahneman, 2003, p. 1454).

The suffering tends to be felt more painful to investors at the waning of a stock price than the happiness which will accrue by the gain of the same value unit in the stock price. In this case, the status quo becomes the reference point. This cognitive process sharply contrasts with the decision-making system of the RAM. In the latter, the works of the preference system remain independent from the setting of reference-point(Kahneman and Tversky, 1979).

In the sub-conscious mind, the attributes of stimulus and conceptual representations tend to impose the framing of senses on the perception of the HCS. In other words, the decisions become not invariant to the shaping of cognitive framing(Tversky and Kahneman, 1986). Johnson and Goldstein (2003) presented a compelling experiment by comparing the proportions of the population enrolled in European countries' organ donation programs. Enrollment was the default case in seven European states. Nonenrolment was the default in four states. Averaging over the opt-out states, the enrollment in the donor program was 97.4 percent. Otherwise(opt-in), it was 18 percent. You have to take explicit action to opt-out. The default forms a cognitive framing and raises the enrollment rate. ‘Invariance is violated in the framing effect'(Kahneman 2003).

When being confronted with complicated details in an uncertain condition, people tend to substitute attributes and make the complex task simpler(Tversky and Kahneman, 1974). In a study by Fritz Strack et al. (1988), college students responded to a survey which included the two following questions in immediate succession: "How happy are you with your life in general?" and "How many dates did you have last month?" The study measured the happiness of life on an 11-point scale. Dating happiness is a specific question, compared with that of life in general. It worked with the same 11-point scale.

The comparison of correlation ratios in two different experiment trials seems to bear a compelling interpretation. In the first trial, they put in place the general question of life happiness before the specific issue of dating happiness. The correlation ratio between the two items was 0.16. In the second trial, they reversed the order of sequence between the two questions. The correlation ratio highly increased to 0.55. When asking for the specific dating happiness ahead of the general life happiness, it formed conceptual priming as the accessible cognition to create judgment heuristics and affected biases when making a judgment on the life happiness in general. However, the reverse order does create attribute substitution when asking a general question before a specific issue.

In List (2002) experiment, traders of sportscards assigned a significantly higher value to a set of ten sportscards labeled "Mint/near mint condition" than to a band that included the same ten cards and three additional cards described as 'poor condition." Jonathan E. Alevy et al. (2003) also confirmed a critical difference (initially suggested by Hsee (1998)) between the prices that people will pay when they see only one of the goods (separate evaluation), or when they price both products at the same time (joint assessment). The items were similar to those used in List's experiment. They observed a predicted violation of dominance in separate evaluations, especially for relatively inexperienced market participants, bidding an average of $4.05 for the small set of cards and only $1.82 for the expansive collection. The violations of dominance completely disappeared in the joint evaluation condition, where the bids for the

small and large sets averaged $2.89 and $3.32, respectively. Alevy et al. (2003) noted that System 1 appears to dominate responses in separate evaluations, whereas System 2 conforms to the dominance rule when given a chance to do so. There was a definite effect of market experience, both in this study and in List (2002): the bids of highly experienced traders also showed violations of monotonicity in separate evaluation, but the effect was much smaller.

III. Preferences of the behavioral decision model

The replacement of the RAM by behavioral approach leads to the change in the model of decision making of individuals. In the dual cognitive system of behavioral approach, the perception-intuition channel of the System 1 offers a more accessible conduit to the human cognitive system than the reasoning channel of the System 2. The consequences of behavioral approach are framing effects in perception, reference dependence in decision making, and heuristics reliance in judgment.

Decision making no longer sustains invariance to the making of perception-framing. Likewise, it does not support independence in the setting of cognitive references. Individuals have to rely on intuitive heuristics in a complex condition when seeking the path to judgments. Individuals' behaviors in the market often violate the scale monotonicity in value measurement. Direct link from state variables like goods to the decision-making by the well-ordered preferences of consumers becomes severed.

The undermining of preference theory means to indicate the breaking of the backbone of the RAM approach. How much is the damage? Can we repair it? How can we rebuild a new decision theory? For this purpose, it seems necessary to interpret the actions according to the behavioral approach into the language, which is audible to the ears of the RAM's decision theory.

According to the RAM approach, human action comes as the outcome of rational choice. There is nothing coincidental in the rational choice theory. However, in the behavioral approach, coincidence is a typical attribute (Rhee, 2018a). From perception to intuition, likewise, with the harboring of framing, reference choice, and heuristics, incidence comes with probability, not an inevitability.

As the unavoidable outgrowth from the coincidence, an economic state tends to bear the attribute of indeterminateness. It contrasts with the determinateness of state variables in the RAM. For instance, an equilibrium state comes as a determinate state in the RAM. However, the equilibrium as a determinate state doesn't exist in the behavioral model. Likewise, the seeking of optimization solutions is not allowed in the BRM.

An incident occurring out of a coincidence makes a legacy, e.g., the preceding incident affects the setting of the path for the following events. Perception, impression, intuition, the shaping of framing, reference choice, the harboring of heuristics all belong to this conceptual class. We may label this attribute as path dependence. Path dependence is a typical attribute of the BRM.

Premise ICP (indeterminateness, coincidence, path-dependence): The analytical behavior of the HCS (human cognitive system) translates as indeterminate, coincidental, and path-dependent.

Three analytical attributes of the BRM, i.e., indeterminateness, coincidence, and path dependence, contrast with the RAM's analytical framework. The RAM contradicts BRM's characteristics to every item. As a consequence, the efficacy of the RAM cannot but be affected severely.

IV. Rational preference ordering

Recognizing the working of the human cognitive system, which is manifested by Premise ICP, undermines the analytical architecture of decision theory in the RAM. In the RAM, the well-ordered consumer's preference lays a firm basis upon which to build consumer choice theory. By this theoretical structure, state variables like goods become directly linked to consumer's actions like a demand. Figure 1 depicts the conceptual image which maps the functional relationship between goods as a state variable and buyer's choice as demand schedule.

The critical feature of the RAM analysis is that we can conduct the analytical reasoning of the consumer's choice consistently and thoroughly employing value measure.[1] It is made possible by the adoption of the well-ordered consumer's preference. The optimization algorithm, which works on the partial ordering, enables the building of demand theory. In a demand function, a functional mapping shoots from the value(price) space to demand space.

Proposition RPO (rational preference ordering): Once we define a goods space, the axiom of choice endorses the existence of rational preference ordering in a set of goods.

Proof: There are n goods. Let C_i be a set of goods in i-th state in the goods space C. There are m i's, i.e., m states for each of n goods. Automobile, cell-phone, padding wear, wine, and baguette bread are some of goods, the number of which goes to n. How many units for each item is another dimension to describe the holding of those goods. i-th state denotes an i-th specific combination of units for each goods item where i = 1, 2, , m.

Therefore, C_i denotes i-th state of a set of n goods in a Euclidean space.

[1] The axiom of choice, i.e., Zorn's lemma and Zermelo's theorem(Dugundji, 1966: ch.2), endorses the legitimacy of the well-ordered buyer's preference ordering.

$$C_i = \{ C_i \in C \mid C \subset \mathbf{R}^n \}.$$

Let's think about C in standard topological space (C, T_S).

D_j denotes the set of n goods in the j-th state (j = 1, 2, , m), the preference ordering of which is compared each other in different states.

$$D_i = \{ D_i \in D \mid D = (\mathbf{R}^n, \lhd) \}$$

, where $\lhd$ denotes the preordering relation in a set. Similarly, we can think about D in a topological space of preference ordering $(D, T_{\lhd})$.

For an intuitive understanding, we may think of a two-dimensional coordinate plane with cell-phone and fashion clothes in each of two axes. We may draw an indifference map in the coordinate plane, which depicts preference ordering relations of a person. (C, T_S) denotes a topological space of the two-dimensional coordinate plane of two goods. $(D, T_{\lhd})$ denotes a topological space of indifference map where two goods are accounted for by the preference ordering of indifference map.

Rational preference ordering means the mapping from (C, T_S) to $(D, T_{\lhd})$. In the example of a two-dimensional (cell-phone and fashion clothes) coordinate plane, the mapping means a drawing of an indifference map in the coordinate plane.

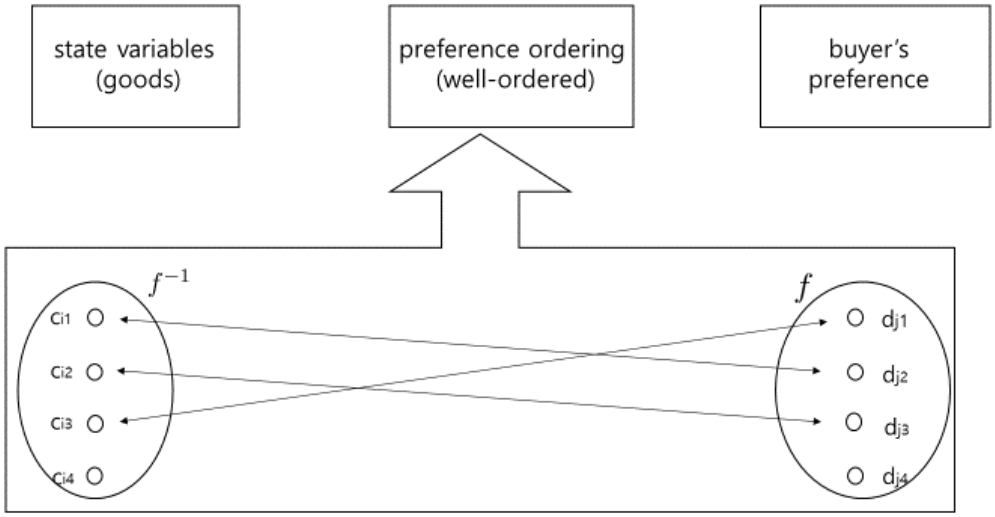
Figure 1 depicts a picture featuring a mapping f from goods space (C, T_S) to a preference space $(D, T_{\lhd})$. It maps c_{i1}, c_{i2}, c_{i3} to d_{i2}, d_{i3}, d_{i1} each, matching a point of domain with a point of the codomain. The preference mapping is drawn as a function in the figure.

$$f : (C, T_S) \rightarrow (D, T_{\lhd}).$$

It is the Axiom of Choice that proves the existence of function mapping f from a goods space (C, T_S) to a preference space $(D, T_{\lhd})$. Zermelo's theorem stipulates that every set can be well-

ordered[2] (Dugundji, 1966: ch. 2, sec. 1).□

Figure 1: Rational agent and preference ordering



The function mapping of Proposition RPO means the determinateness of mind in the decision-making of a person in one's preference revealing. There is no indeterminateness. However, it is a rare case in real life. It is not rational counting, but the HCS that dictates the decisions on preference revealing. The perception prompts coincidentally. Image framing is somewhat indeterminate. So the reference-point or heuristics anchoring is. Once they are determined, the ensuing decisions become reliant on them. As Premise ICP stipulates, there is path dependence in the cognitive system.

V. Untenable preference function

The recognition of the HCS invites a critical problem to the mapping of one's preference revealing. The mapping from goods space to the space of buyer's preference has to go through the HCS. Figure 2 depicts two mappings; the first mapping g from a goods space X to a cognitive space Ξ and the second h from a cognitive space Ξ to preference space Y. Neither of these mappings is a function mapping. The function mapping has to go uniquely from the domain to codomain. For a point in a domain $x_i \in X$, there has to be *one and only one* destination point in a codomain $\xi_i \in \Xi$.

[2] A binary relation in a set is called a preordered set if it is reflexive and transitive (Dugundji, 1966: ch. 2, sec. 1)

Figure 2 shows a multi-headed mapping in mapping g as well as in mapping h. For instance, the mapping g shoots x_1 in a goods space X to three images ξ_2, ξ_3, ξ_4 in a cognitive space Ξ. The perception-intuition or impression coincidentally occurs so that it is not determinate nor unique. It is indeterminate.

The mapping h shoots ξ_1 in the cognitive space Ξ to three images y_1, y_2, y_4 in the preference space Y. The perception-intuition, combined with reasoning, never determines preference mapping uniquely or determinately. We rely on reference points. The anchoring of framing or heuristics, upon which we rely for the decision-making, is never a determinate process.

Therefore, we are not sure which ξ_i will turn out as its image in response to xi's incidence in the mapping g. The story of uncertain turnout is the same in case of the mapping h.

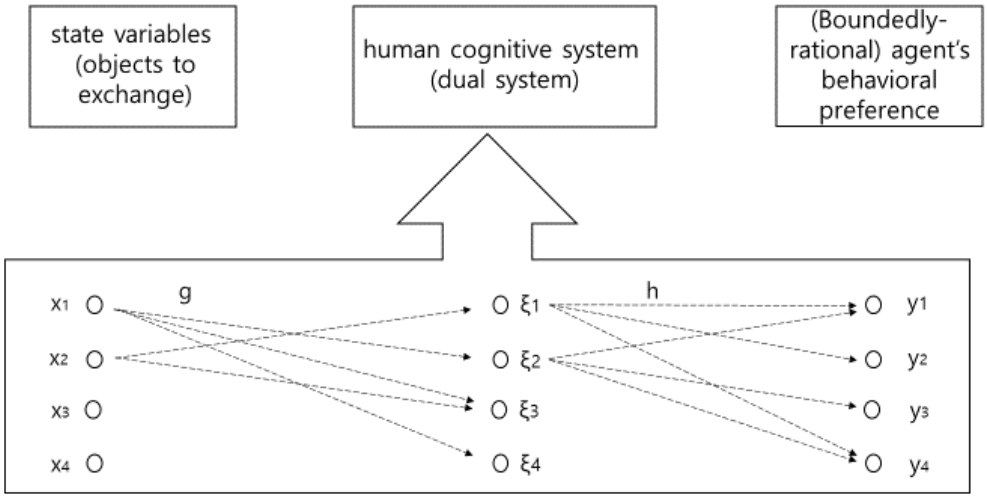
Proposition UPF (untenable preference function): Under the assumption HCS, the forming of a preference function from goods space is unsustainable.

Proof: The mapping g from goods space X to the cognitive space Ξ is not unique. Hence, mapping g is not a function. Neither the mapping h from the cognitive space Ξ to the preference space Y is unique. Therefore, the composite mapping φ, where $\varphi := g \circ h$, is not a function□

Note that the composite mapping φ is preference mapping.

At an instance x_i of a state in goods space (X, T_S), we are not certain about the outcome in the preference decision y_i. It could be y_1 or y_2 or y_4, as Figure 2 reveals. The preference response is not unique. It happens coincidentally; your fashion garment may look charming sometimes, but not look so sometime later. Preference revealing is indeterminate. It raises a critical problem, i.e., the metrizability of preference.

Figure 2: Human cognitive system and (boundedly-rational) agent's behavioral preference



Due to the uncertain outcome in preference, the preference outcome y_i is not bounded. We never know the vagary of sensory response when encountering the same state x_i in goods space. It could mean that no subset in order topology $(Y, T_{\prec})$ is compact unless we discard the assumption of the HCS.

Proposition UCompactPS (untenable compactness of the preference space): If we assume the HCS, no subset in the order topology in preference space $(Y, T_{\prec})$ is compact.

Proof: Due to the uncertain outcome in the preference under the assumption of the HCS, the preference outcome y_i is not bounded. It means no subset in order topology $(Y, T_{\prec})$ is compact. □

This paper aims to prove that we cannot use the function mapping φ of goods space to describe the preference behavior in preference space $(Y, T_{\prec})$. Now, we are ready to do so.

Proposition EVT (extreme value theorem): Let $\varphi : X \rightarrow Y$ be continuous, where Y is an ordered set in the order topology, If X is compact, then there exist points c and d in X such that $\varphi(c) \leq \varphi(x) \leq \varphi(d)$ for every $x \in X$.

Proof: Munkres (2018: the proof of Theorem 27.4, p. 174).□

Proposition EVT states that for any instance of x_i in goods space, the image $\varphi(x_i)$ of a continuous function φ is bounded by two determinate values $\varphi(c) \leq \varphi(d)$. However, Premise ICP states the indeterminateness of image value $\varphi(x_i)$. In other words, we cannot find any determinate values $\varphi(c)$, $\varphi(d)$ as a boundary for $\varphi(x_i)$.

What Proposition EVT implies is that we cannot evoke the function mapping φ of goods space to describe the preference behavior in preference space $(Y, T_{\prec})$.

Remark IFMPB (Illegitimate Function Mapping for Preference Behavior): The function mapping φ of goods space cannot be adopted to describe the preference behavior in preference space $(Y, T_{\prec})$.

However, the problem seems to go deeper than that. It becomes the problem of metrizability in preference space $(Y, T_{\prec})$. In this regard, let us probe the subsets' completeness in preference space $(Y, T_{\prec})$. The uncertain outcome of preference under the assumption HCS renders the completeness in preference space $(Y, T_{\prec})$ untenable.

Proposition UCompletePS (untenable completeness of preference space): Under the assumption HCS, no Cauchy sequence in the preference space converges.

Proof: Cauchy sequence has to converge to establish the subsets Y's completeness in preference space $(Y, T_{\prec})$. For any ε, there exists N such that if $n, m > N$, $d(y_n, y_m) < \varepsilon$, where d denotes the symbol of metric and $d(\ ,\ )$ means distance. This convergence is impossible due to the indeterminate outcome of y_i. Hence, the preference space $(Y, T_{\prec})$ is not complete.□

For instance, a particular seizure of ungrounded fear or incomprehensible distrust may cause the rise of an isolated extreme case of preference incident, which blocks the possibility of the completeness of any subset in preference space $(Y, T_{\prec})$. The non-compliance with the completeness gives rise to preference space's metrizability problem $(Y, T_{\prec})$ (Dugundji, 1966: p. 194). Premise ICP eventually leads to the repudiation of the metrizability of preference space $(Y, T_{\prec})$ (Rhee 2018a).

VI. Post-script

The untenable metrizability of preference space $(Y, T_{\prec})$ reminds us of Hume's divide (Rhee 2018e). The core argument of Hume's epistemology is that rational reasoning, like value theory, is reliable only when the principle of the uniformity of nature (PUN in short) holds. However, Premise ICP refuses to accept the metrizability of preference space and rejects PUN's possibility. How can we navigate for the making of decisions if the value measure is not available? Hume's answer was the experience. Premise ICP opens the gateway to the study of the empiricism approach in economics.

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